EDO – CÁLCULO NUMÉRICO

$$\begin{cases} y` = 2xy \\ y(0) = 1 \end{cases}$$

$$Aproximar \; y(0.5) = ?$$

$$\frac{dy}{dx} = 2xy$$

$$\frac{dy}{y} = 2x\,dx$$

$$\ln(y) = x^2 + C$$

$$\ln(1) = 0^2 + C$$

$$C = 0$$

$$\ln(y) = x^2$$

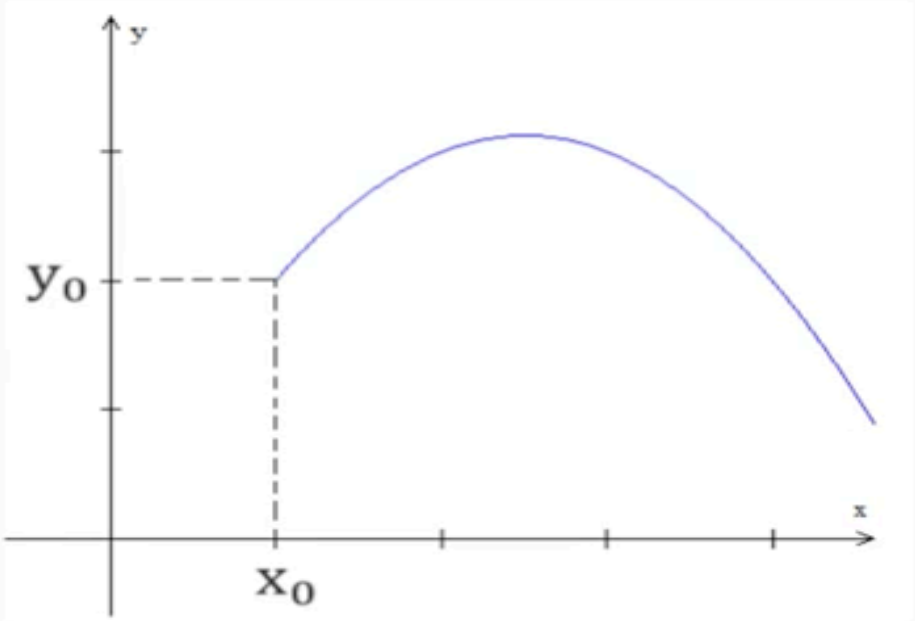
$$e^{\ln(y)} = e^{x^2}$$

$$y = e^{x^2}$$

$$y = e^{0.5^2}$$

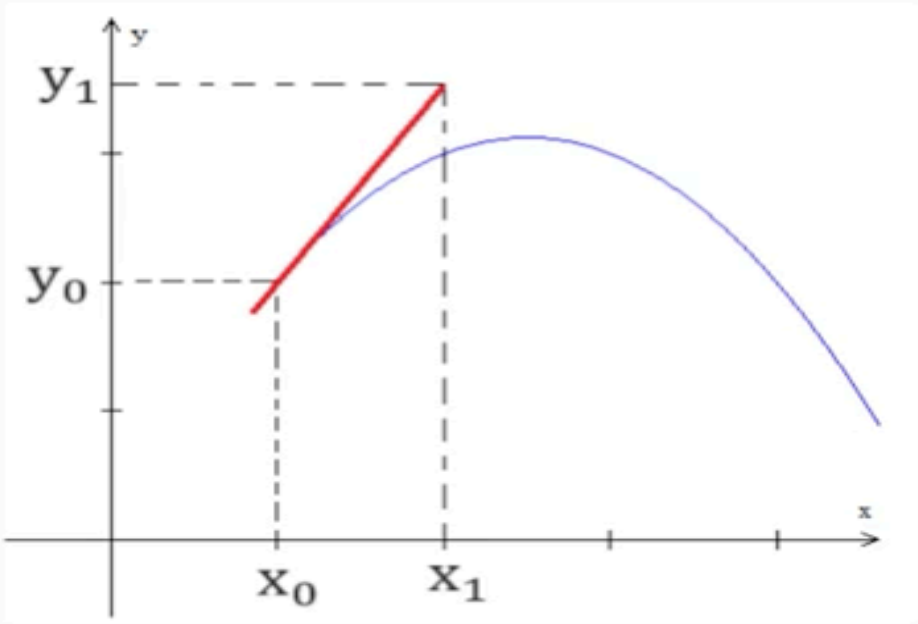
$$y = 1{,}284025$$

Método de Euler



$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

Método de Euler



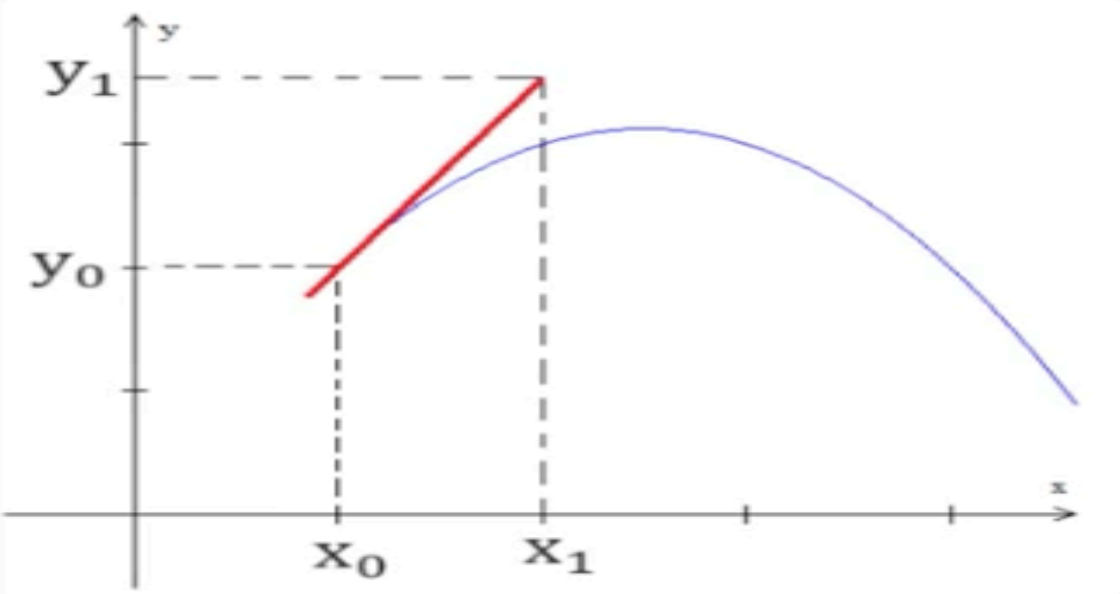
$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

$$h = x_1 - x_0$$

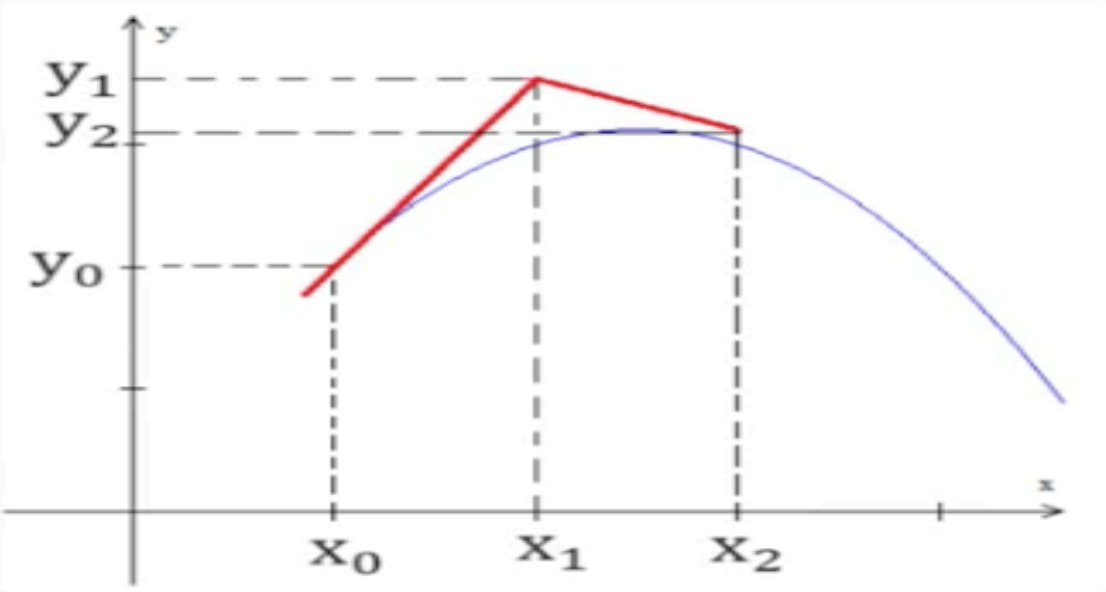
$$\frac{y_1 - y_0}{h} \approx y'(x_0)$$

$$y_1 - y_0 \approx h y'(x_0)$$

$$y_1 \approx y_0 + h y'(x_0)$$

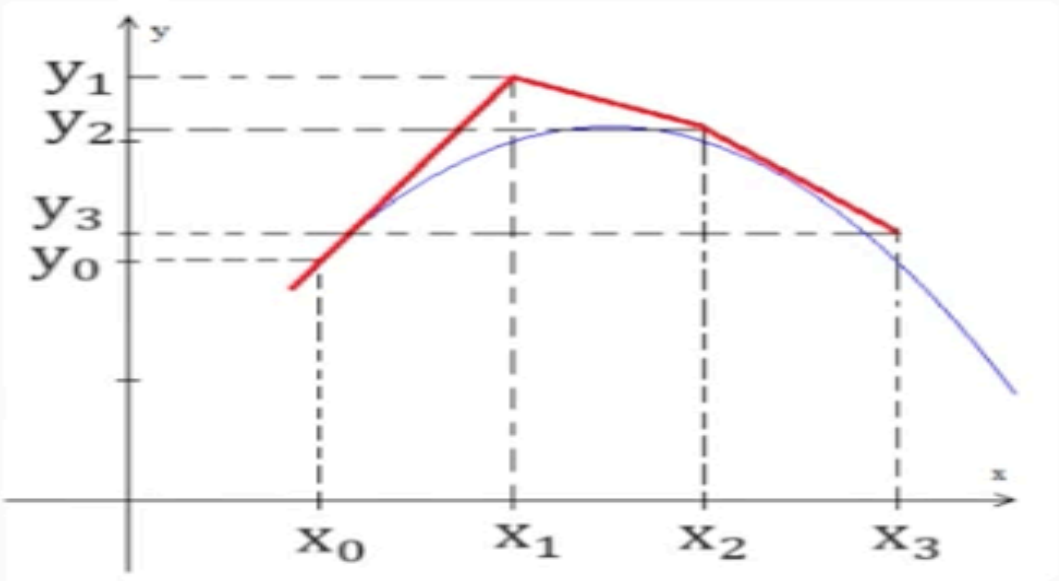


$$y_1 = y_0 + hy'(x_0)$$



$$y_1 = y_0 + hy'(x_0)$$

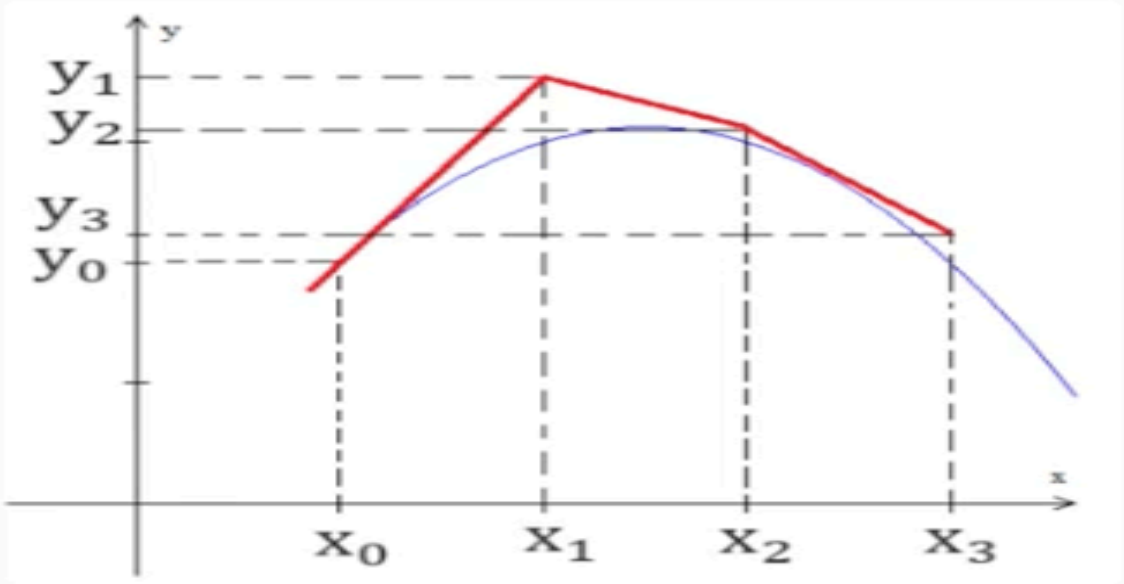
$$y_2 = y_1 + hy'(x_1)$$



$$y_1 = y_0 + hy'(x_0)$$

$$y_2 = y_1 + hy'(x_1)$$

$$y_3 = y_2 + hy'(x_2)$$



$$y_1 = y_0 + hy'(x_0)$$

$$y_2 = y_1 + hy'(x_1)$$

$$y_3 = y_2 + hy'(x_2)$$

$$y_4 = y_3 + hy'(x_3)$$

$$\begin{cases} y` = 2xy \\ y(0) = 1 \end{cases}$$

$$Aproximar\ y(0.5) = ?$$

$$x_o = 0$$
$$y_o = 1$$
$$h = 0.1$$

$$y_1 = y_0 + h[f(x_0, y_0)]$$
$$y_1 = 1 + 0{,}1[2.(0).(1)]$$
$$y_1 = 1 + 0 = 1$$

$$y_2 = y_1 + h[f(x_1, y_1)]$$
$$y_2 = 1 + 0{,}1[2.(0{,}1).(1)]$$
$$y_1 = 1 + 0{,}02 = 1{,}02$$

i	x	y	f'(x,y)	yn
0	0	1	0	1
1	0,1	1	0,2	1,02
2	0,2	1,02	0,408	1,0608
3	0,3	1,0608	0,63648	1,124448
4	0,4	1,124448	0,899558	1,214404
5	0,5	1,214404		

Runge-Kutta 2ª ordem

$$u_{i+1} = u_i + \frac{h}{2}[K_1 + K_2] \qquad i = 0, 1, \ldots, N-1$$

$$K_1 = f(x_i, u_i)$$

$$K_2 = f(x_i + h, u_i + h.K_1)$$

i	x	y	k1	xi+h	yi+h*k1	k2	yn
0	0	1	0	0,1	1	0,2	1,01
1	0,1	1,01	0,202	0,2	1,0302	0,41208	1,040704
2	0,2	1,040704	0,416282	0,3	1,082332	0,649399	1,093988
3	0,3	1,093988	0,656393	0,4	1,159627	0,927702	1,173193
4	0,4	1,173193	0,938554	0,5	1,267048	1,267048	1,283473
5	0,5	1,283473					

Runge-Kutta 4^a ordem

$$u_{i+1} = u_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4] \qquad i = 0, 1, \ldots, N-1$$

$$K_1 = h \;.\; f(x_i, u_i)$$

$$K_2 = h \;.\; f(x_i + \frac{h}{2}, u_i + \frac{K_1}{2})$$

$$K_3 = h \;.\; f(x_i + \frac{h}{2}, u_i + \frac{K_2}{2})$$

$$K_4 = h \;.\; f(x_i + h, u_i + K_3)$$

I	X	Y	k1	Xi+h/2	Yi+K1/2	K2
			h*f(xi,yi)			h*f((xi+h/2);(yi+K1/2))
0,0000	0,0000	1,0000	0,0000	0,0500	1,0000	0,0100
1,0000	0,1000	1,0101	0,0202	0,1500	1,0202	0,0306
2,0000	0,2000	1,0408	0,0416	0,2500	1,0616	0,0531
3,0000	0,3000	1,0942	0,0657	0,3500	1,1271	0,0789
4,0000	0,4000	1,1736	0,0939	0,4500	1,2205	0,1098
5,0000	0,5000	1,2841				

yi+k2/2	k3	Xi+h	yi+k3	k4	yn
	h*f((xi+h/2);(yi+k2/2))			h*f((xi+h);(yi+k3))	
1,0050	0,0101	0,1000	1,0101	0,0202	1,0101
1,0254	0,0308	0,2000	1,0409	0,0416	1,0408
1,0674	0,0534	0,3000	1,0942	0,0657	1,0942
1,1337	0,0794	0,4000	1,1736	0,0939	1,1736
1,2285	0,1106	0,5000	1,2842	0,1284	1,2841

$y' = x+2xy$
$h = 0.1$
$y_0 = 1$
$x_0 = 0.5$

$$u_{i+1} = u_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$