$$f(x) \equiv (x-r)^m \cdot q(x) \Rightarrow f'(x) \equiv m \cdot (x-r)^{m-1} \cdot q(x) + (x-r)^m \cdot q'(x)$$

$$\Rightarrow f'(x) \equiv m \cdot (x-r)^{m-1} \cdot q(x) + (x-r)^m \cdot q'(x)$$

Portanto, temos: $f'(x) \equiv (x-r)^{m-1} \cdot \left[m \cdot q(x) + (x-r)^m \cdot q'(x) \right]$ e, como

$m \cdot q(r) + (x-r)^m \cdot q'(r) = m \cdot q(r) \neq 0$, temos que r é raiz de multiplicidade m–1 de $f'(x) = 0$.

Corolário 1:

Se r é raiz de multiplicidade m da equação $f(x) = 0$, então r é raiz de:

$$f^{(1)}(x) = 0,\ f^{(2)}(x) = 0,\ f^{(3)}(x) = 0, \ldots, f^{(m-1)}(x) = 0$$

com multiplicidade m–1, m–2, m–3, …,1, respectivamente, e r não é raiz de $f^{(m)}(x) = 0$.

Corolário 2:

Se r é raiz das equações $f(x) = 0, f^{(1)}(x) = 0,\ f^{(2)}(x) = 0,\ f^{(3)}(x) = 0, \ldots, f^{(m-1)}(x) = 0$ e r não é raiz da equação $f^{(m)}(x) = 0$, então a multiplicidade de r em $f(x) = 0$ é m.

Resumindo:

"A condição necessária e suficiente para que um número r seja raiz com multiplicidade m de uma polinomial $f(x) = 0$ é que r seja raiz das funções $f(x), f^{(1)}(x),\ f^{(2)}(x), \ldots, f^{(m-1)}(x)$ e não seja raiz $f^{(m)}(x)$".

Questões Resolvidas

01) Determinar a derivada das seguintes funções:

1) $f(x) = x^2 \cdot \sqrt{1+x^3}$

$$f'(x) = u'(x).v(x) + u(x).v'(x)$$

$$f'(x) = 2x \cdot \sqrt{1+x^3} + x^2 \cdot \frac{3x^2}{2 \cdot \sqrt{1+x^3}}$$

$$f'(x) = 2x \cdot \sqrt{1+x^3} + \frac{3x^4}{2 \cdot \sqrt{1+x^3}}$$

$$v(x) = \sqrt{1+x^3} = (1+x^3)^{\frac{1}{2}} \Rightarrow v'(x) = \frac{1}{2} \cdot (1+x^3)^{\frac{1}{2}-1}.3x^2 \Rightarrow$$

$$v'(x) = \frac{1}{2} \cdot (1+x^3)^{-\frac{1}{2}}.(0+3x^2) \Leftarrow \frac{1}{2} \cdot \frac{1}{(1+x^3)^{\frac{1}{2}}} \cdot 3x^2 \Rightarrow$$

$$v'(x) = \frac{3x^2}{2.\sqrt{1+x^3}}$$

2) $f(x) = \dfrac{1+\text{sen x}}{1-\text{sen x}}$

$$u(x) = 1 + senx \Rightarrow u'(x) = \cos x$$

$$v(x) = 1 - senx \Rightarrow v'(x) = -\cos x$$

$$f'(x) = \frac{u'(x).v(x) - u(x).v'(x)}{\left[v(x)\right]^2} \Rightarrow f'(x) = \frac{\cos x.(1 - sen\,x) - \left[(1+sen\,x)\right].(-\cos x)}{\left[(1-sen\,x)\right]^2}$$

$$f'(x) = \frac{\cos x - \cancel{\cos x.sen\,x} + \cos x + \cancel{\cos x.sen\,x}}{\left[(1-sen\,x)\right]^2} \Rightarrow f'(x) = \frac{2.\cos x}{\left[(1-sen\,x)\right]^2}$$

3) $f(x) = \dfrac{\text{sen x} + \cos x}{\text{sen x} - \cos x}$

$$f'(x)=\frac{(\cos x-sen\,x)\cdot(sen\,x-\cos x)-\left[(sen\,x+\cos x)\cdot(\cos x+sen\,x)\right]}{\left[sen\,x-\cos x\right]^2}$$

$$f'(x)=\frac{\cos x.sen\,x-\cos^2 x-sen^2x+sen\,x.\cos x-\left[sen\,x.\cos x+sen^2x+\cos x^2+\cos x.sen\,x\right]}{\left[sen\,x-\cos x\right]^2}$$

$$f'(x)=\frac{\cancel{\cos x.sen\,x}-\cos^2 x-sen^2x\cancel{+sen\,x.\cos x}\cancel{-sen\,x.\cos x}-sen^2x-\cos x^2\cancel{-\cos x.sen\,x}}{\left[sen\,x-\cos x\right]^2}$$

$$f'(x)=\frac{-\cos^2 x-sen^2x-sen^2x-\cos x^2}{\left[sen\,x-\cos x\right]^2}\Rightarrow\frac{-2.\cos^2 x-2.sen^2x}{\left[sen\,x-\cos x\right]^2}\Rightarrow\frac{-2.(\cos^2 x+sen^2x)}{\left[sen\,x-\cos x\right]^2}$$

$$f'(x)=\frac{-2.1}{\left[sen\,x-\cos x\right]^2}\Rightarrow f'(x)=\frac{-2}{\left[sen\,x-\cos x\right]^2}$$

4) $f(x)=\dfrac{\text{tg x}}{\text{sen x}+\cos\text{x}}$

$u(x)=tg\,x\Rightarrow u'(x)=\sec^2 x$

$v(x)=sen\,x+\cos x\Rightarrow v'(x)=\cos x-sen\,x$

$$f'(x)=\frac{u'(x).v(x)-u(x).v'(x)}{\left[v'(x)\right]^2}\Rightarrow f'(x)=\frac{sen^2x.(\cos x+sen\,x)-tg\,x.(\cos x-sen\,x)}{\left[sen\,x-\cos x\right]^2}$$

5) $f(x)=\sec\text{x}\cdot\ln\text{x}$

$f'(x)=u'(x).v(x)+u(x).v'(x)$

$u(x)=\sec x\Rightarrow u'(x)=tg\,x.\sec x$

$v(x)=\ln x\Rightarrow v'(x)=\dfrac{1}{x}$

$$f'(x)=tg\,x\cdot\sec x\cdot\ln x+\sec x\cdot\frac{1}{x}$$

$$f'(x)=\sec x\cdot\left(tg\,x\cdot\ln x+\frac{1}{x}\right)$$

6) $f(x)=4\cdot\sec\text{x}+3\cdot\text{cossec x}$

$u(x)=\sec x\Rightarrow u'(x)=tg\,x\cdot\sec x$

$v(x)=\text{cos}\sec x\Rightarrow v'(x)=-\text{cotg}\,x\cdot\text{cos}\sec x$

$$f'(x)=u'(x)\cdot v(x)+u(x)\cdot v(x)$$

$$f'(x)=4\cdot tg\,x\cdot\sec x+3\cdot(-\text{cotg}\,x\cdot\text{cos}\sec x)$$

$$f'(x)=4\cdot tg\,x\cdot\sec x-3\cdot\text{cotg}\,x\cdot\text{cos}\sec x$$

7) $f(x)=\text{e}^{-\text{cossec}^3\text{x}}$

$$f'(x)=\text{e}^{-\text{cossec x}^3}.-(-\text{cossec x}^3\cdot\text{cotg x}^3\cdot 3\text{x}^2)$$

$$f'(x)=3\text{x}^2.\text{e}^{-\text{cossec x}^3}.(\text{cossec x}^3\cdot\text{cotg x}^3)$$

8) $f(x)=\cos\text{x}\cdot\text{cotg x}$

$$f'(x)=u'(x)\cdot v(x)+u(x)\cdot v'(x)$$

$$f'(x)=-sen\,x\cdot\text{cotg}\,x+\cos x\cdot(-\text{cos}\sec^2 x)\Rightarrow-\cancel{sen\,x}\cdot\frac{\cos x}{\cancel{sen x}}+\cos x\cdot\left(\frac{1}{sen^2x}\right)\Rightarrow-\cos x+\cos x\cdot\left(\frac{1}{sen^2x}\right)$$

$$f'(x)=-\cos x-\frac{\cos x}{sen\,x}\cdot\frac{1}{sen\,x}\Rightarrow f'(x)=-\cos x-\text{cotg}\,x.\text{cos}\sec x$$

9) $f(x)=(\text{x}+\text{cossec x})\cdot\ln\text{x}$

$u(x)=x\Rightarrow u'(x)=\left[1+\left(-\text{cotg}\,x.\text{cossec}\,x\right)\right]$

$v(x)=\ln x\Rightarrow v'(x)=\dfrac{1}{x}$

$f'(x) = u'(x).v(x) + u(x).v'(x)$

$f'(x) = (1 - \text{cotg } x.\text{cossec } x) \cdot \ln x + (x + \text{cossec } x) \cdot \frac{1}{x}$

$f'(x) = (1 - \text{cotg } x.\text{cossec } x) \cdot \ln x + \left(\frac{x}{x} + \text{cossec } x \cdot \frac{1}{x} \right) \Rightarrow \boxed{f'(x) = (1 - \text{cotg } x.\text{cossec } x) \cdot \ln x + 1 + \frac{1}{x} \cdot \text{cossec } x}$

10) $f(x) = \frac{x^2 \cdot \text{sen } x}{e^x}$

$u(x) = x^2.sen\,x \Rightarrow u'(x) = 2x.sen\,x + x^2.\cos x$

$v(x) = e^x \Rightarrow v'(x) = e^x$

$f'(x) = \frac{u'(x) \cdot v(x) - u(x).v'(x)}{[v(x)]^2}$

$f'(x) = \frac{(2x \cdot sen\,x + x^2 \cdot \cos x) \cdot e^x - x^2 \cdot sen\,x \cdot e^x}{[e^x]^2} = \frac{e^x \cdot x \cdot [2 \cdot sen\,x + x \cdot \cos x - x \cdot sen\,x]}{[e^x]^2}$

$\boxed{f'(x) = \frac{x \cdot [2 \cdot sen\,x + x \cdot \cos x - x \cdot sen\,x]}{e^x}}$

11) $f(x) = \frac{e^{x^2+1}}{\cos x}$

$u(x) = e^{x^2+1} \Rightarrow u'(x) = e^{x^2+1}.2x$

$v(x) = \cos x \Rightarrow v'(x) = -senx$

$f'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{[v(x)]^2} \Rightarrow \frac{(2x \cdot e^{x^2+1}) \cdot \cos x + e^{x^2+1} \cdot sen\,x}{[\cos x]^2} \Rightarrow \frac{(2x.e^{x^2+1}).\cos x + e^{x^2+1} \cdot sen\,x}{\cos^2 x}$

$\boxed{f'(x) = \frac{e^{x^2+1}(2x \cdot \cos x + sen\,x)}{\cos^2 x}}$

12) $f(x) = \log_3^{\text{sen } x}$

$f(x) = \log_a^x \Rightarrow f'(x) = \frac{1}{\ln(a)}$

$f'(x) = \frac{\cos x}{sen\,x \cdot \ln 3}$

$\boxed{f'(x) = \frac{\text{cotg } x}{\ln 3}}$

* A derivada de $sen\,x$ é $\cos x$.

* $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x \cdot \ln a}$

* $f(x) = \log_a^x \Rightarrow f'(x) = \frac{1}{x \cdot \ln a}$

13) $f(x) = e^{\text{sen}(x^2+5x+1)}$

$f'(x) = e^{sen(x^2+5x+1)}.\cos(x^2 + 5x + 1).(2x + 5)$

$\boxed{f'(x) = (2x + 5).e^{sen(x^2+5x+1)}.\cos(x^2 + 5x + 1)}$

Derivada da parte interna

$= x^2 + 5x + 1$

$= 2x + 5.1 + 0$

$= 2x + 5$

14) $f(x) = \log_2^{[\sec(x^2-1)]}$ $\quad y' = \frac{tg(x^2 - 1).\sec(x^2 - 1).2x}{\sec(x^2 - 1) \cdot \ln 2} \Rightarrow \boxed{y' = \frac{2x.tg(x^2 - 1)}{\ln 2}}$

15) $f(x) = \sec(\sqrt[3]{x})$

$f'(x) = 3.\left(\sec\sqrt{x}\right)^{3-1}$ → Deriva a parte interna e multiplica por $3.\left(\sec\sqrt{x}\right)^2$

$f'(x) = 3.\left(\sec\sqrt{x}\right)^2 .\sec\sqrt{x}.\ tgx\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$ → Deriva a parte interna; → Deriva do arco

- A derivada de $\left(\sec\sqrt{x}\right) \Rightarrow \sec\sqrt{x}\cdot tg\sqrt{x}$
- A derivada de $\sqrt{x} \Rightarrow \frac{1}{2.\sqrt{x}}$

$f'(x) = 3.\sec\sqrt[2]{x}.\sec\sqrt{x}.\ tgx\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{x}}\cdot 3.\sec\sqrt[3]{x}.\ tgx\sqrt{x}$

$f'(x) = \frac{3}{2\sqrt{x}}\cdot\sec\sqrt[3]{x}\cdot\ tgx\sqrt{x}$

16) $f(x) = \sqrt{\text{cossec } 2\theta}$

$f'(x) = (\text{cossec } 2\theta)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}\cdot(\text{cossec } 2\theta)^{\frac{1}{2}-1} \Rightarrow \frac{1}{2}\cdot(\text{cossec } 2\theta)^{-\frac{1}{2}}\cdot -\text{cotg } 2\theta\cdot\text{cossec } 2\theta$

$f'(x) = \frac{1}{2}\cdot(\text{cossec } 2\theta)^{-\frac{1}{2}}\cdot -\text{cotg } 2\theta\cdot\text{cossec } 2\theta\cdot 2.(1) \Rightarrow \frac{1}{\not 2}\cdot(\text{cossec } 2\theta)^{-\frac{1}{2}}\cdot -\text{cotg } 2\theta\cdot\text{cossec } 2\theta\cdot\not 2$

$f'(x) = (\text{cossec } 2\theta)^{-\frac{1}{2}}\cdot -\text{cotg } 2\theta\cdot(\text{cossec } 2\theta)^1$

$f'(x) = (\text{cossec } 2\theta)^{-\frac{1}{2}+1}\cdot -\text{cotg } 2\theta \Rightarrow (\text{cossec } 2\theta)^{\frac{1}{2}}\cdot -\text{cotg } 2\theta \Rightarrow f'(x) = \sqrt{\text{cossec } 2\theta}\cdot -\text{cotg } 2\theta$

$f'(x) = -\text{cotg } 2\theta\cdot\sqrt{\text{cossec } 2\theta}$

17) $f(x) = \left[2^x\right]^{\cos x}$

$f'(x) = \left[u(x)\right]^{v(x)}\cdot\left[v'(x)\cdot \ln\left[u(x)\right] + v(x)\cdot\frac{u'(x)}{u(x)}\right] \Rightarrow \left[2^x\right]^{\cos x}\cdot\left[-sen\,x\cdot\ln\left[2^x\right] + \cos x\cdot\frac{\not{\left[2^x\right]}\cdot \ln(2)}{\not{\left[2^x\right]}}\right]$

$f'(x) = \left[2^x\right]^{\cos x}\cdot\left[-x\cdot sen\,x\cdot\ln 2 + \cos x\cdot\ln(2)\right]$

18) $\text{f(x)} = \text{cotg}^5\left(e^{x^2+1}\right) \Rightarrow \left[cotg\left(e^{x^2+1}\right)\right]^5$

$f'(x) = 5\cdot\left[cotg\left(e^{x^2+1}\right)\right]^{5-1}\cdot -\text{cossec}^2\left(e^{x^2+1}\right)\cdot\left(e^{x^2+1}\right)\cdot 2x + 0$

$f'(x) = 5\cdot\left[cotg\left(e^{x^2+1}\right)\right]^4\cdot -\text{cossec}^2\left(e^{x^2+1}\right)\cdot\left(e^{x^2+1}\right)\cdot 2x \Rightarrow 5\cdot 2x\cdot\left(e^{x^2+1}\right)\cdot\left[cotg\left(e^{x^2+1}\right)\right]^4\cdot -\text{cossec}^2\left(e^{x^2+1}\right)$

$f'(x) = -10x\cdot\left(e^{x^2+1}\right)\cdot cotg^4\left(e^{x^2+1}\right)\cdot\text{cossec}^2\left(e^{x^2+1}\right)$

19) $f(x) = \text{sen}\left(\frac{x+1}{x-1}\right)$

- 1º deriva a função $\left[sen\left(\frac{x+1}{x-1}\right)\right]$.
- 2º deriva o arco $\left(\frac{x+1}{x-1}\right) \Rightarrow \frac{u'(x)\cdot v(x) - u(x)\cdot v'(x)}{\left[v(x)\right]^2}$.
- $\frac{1.(x-1)-(x+1).1}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$f'(x) = \cos.\left(\frac{x+1}{x-1}\right)\cdot -\frac{2}{(x-1)^2}$

$f'(x) = -\frac{2}{(x-1)^2}\cdot\cos.\left(\frac{x+1}{x-1}\right)$

20) $f(x) = \left[x \cdot \text{sen}\, 2x + \text{tg}^4(x^7)\right]^5$

$$f'(x) = 5 \cdot \left[x \cdot sen\,2x + tg^4(x^7)\right]^4 \cdot \left[sen\,2x + x \cdot \cos 2x \cdot 2 + 4tg^3 x^7 \cdot \sec x^7 \cdot 7x^6\right]$$

$$f'(x) = 5 \cdot \left[x \cdot sen\,2x + tg^4(x^7)\right]^4 \cdot \left[sen\,2x + 2x \cdot \cos 2x + 28x^6 \cdot tg^3 x^7 \cdot \sec x^7\right]$$

02) Seja $y = e^{2x}$. Verifique que $\frac{d^2y}{dx^2} - 4y = 0$

$$\frac{dy}{dx} = e^{2x}.2 \Rightarrow 2.e^{2x} \Rightarrow \frac{dy}{dx} = 2.e^{2x}.2 \Rightarrow 4.e^{2x} \Rightarrow 4.e^{2x} - 4y = 0 \Rightarrow \cancel{4.e^{2x}} - \cancel{4.e^{2x}} = 0$$

03) Seja $y = \cos(w \cdot t)$, *w* constante. Verifique que $\frac{d^2y}{dt^2} + w^2 \cdot y = 0$.

Solução:

$z = w.\,t$

$$z' = w.1 \Rightarrow w \Rightarrow \frac{dy}{dx} = -sen(w.\,t).w \Rightarrow \frac{d^2y}{dx^2} = -\cos(w.\,t).w.w + \cancel{\left[sen(w.\,t)\right]}.0$$

$$\frac{d^2y}{dx^2} = -w^2.\cos(w.\,t) \Rightarrow \frac{d^2y}{dx^2} + w^2 y = 0 \Rightarrow -w^2.\cos(w.\,t) + w^2.y = 0 \Rightarrow \cancel{-w^2.\cos(w.\,t)} + \cancel{w^2.\left[\cos(w.\,t)\right]} = 0$$

$\underline{0 = 0}$

04) Encontre as funções custo médio e custo marginal. Para as funções abaixo:

Solução:

a) $C(x) = 3700 + 5x - 0{,}04x^2 + 0{,}0003x^3$.

$$C(M) = \frac{c(x)}{x} \Rightarrow \frac{3700}{x} + \frac{5x}{x} - \frac{0{,}04x^2}{x} + \frac{0{,}0003x^3}{x} \Rightarrow C(M) = \frac{3700}{x} + 5 - 0{,}04x + 0{,}0003x^2$$

$$C(x) = 3700 + 5x - 0{,}04x^2 + 0{,}0003x^3 \Rightarrow C'(x) = 5 - 0{,}08x + 0{,}0009x^2$$

b) $C(x) = 339 + 25x - 0{,}09x^2 + 0{,}0004x^3$.

$$C(M) = \frac{c(x)}{x} \Rightarrow \frac{339}{x} + \frac{25x}{x} - \frac{0{,}09x^2}{x} + \frac{0{,}0004x^3}{x}$$

$$C(M) = \frac{339}{x} + 25 - 0{,}09x + 0{,}0004x^2$$

$$C(x) = 339 + 25x - 0{,}09x^2 + 0{,}0004x^3$$

$$C'(x) = 25 - 0{,}18x + 0{,}0012x^2$$

05) Um fabricante de pequenos motores estima que o custo da produção de x motores por dia é dado por $C(x) = 100 + 50x + \frac{100}{x}$, compare o custo marginal da produção de 5 motores. Com o custo para produção do sexto motor.

Solução:

$$C(x) = 100 + 50x + \frac{100}{x} \Rightarrow C'(x) = 50 - \frac{100}{x}$$

a) $C'(5) = 50 - \frac{100}{5^2} \Rightarrow 50 - \frac{100}{25} \Rightarrow 50 - 4{,}00 \Rightarrow C'(5) = R\$\ 46{,}00$

b) $C'(6) = 50 - \frac{100}{6^2} \Rightarrow 50 - \frac{100}{36} \Rightarrow 50 - 2{,}77 \Rightarrow C'(6) = R\$\ 47{,}23$