Limites Fundamentais e Trigonométricos

(1) $\text{sen}^2 x + \cos^2 x = 1$

(2) $1 + \text{tg}^2 x = \sec^2 x$

(3) $1 + \text{cotg}^2 x = \text{cosec}^2 x$

(4) $\text{sen}^2 x = 1/2\ (1 - \cos 2x)$

(5) $\cos^2 x = 1/2\ (1 + \cos 2x)$

(6) $\text{sen}\ 2x = 2\ \text{sen}\ x\ \cos x$

(7) $\text{sen}\ x\ \cos y = 1/2\ [\text{sen}\ (x - y) + \text{sen}\ (x + y)]$

(8) $\text{sen}\ x\ \text{sen}\ y = 1/2\ [\cos (x - y) - \cos\ (x + y)]$

(9) $\cos x\ \cos y = 1/2\ [\cos (x - y) + \cos\ (x + y)]$

$\lim_{x \to 0} \frac{\operatorname{sen} x}{x}$ é igual a 1.

$\lim_{x \to 0} \frac{\operatorname{sen} 2x}{x}$

Neste exemplo, $u = 2x$ e $u \to 0$ quando $x \to 0$. Portanto,

$$\lim_{x \to 0} \frac{\operatorname{sen} 2x}{x} = \lim_{u \to 0} \frac{\operatorname{sen} u}{u/2} = 2 \lim_{u \to 0} \frac{\operatorname{sen} u}{u} = 2 \cdot 1 = 2.$$

$$\lim_{x \to 0} \frac{sen\ 3x}{sen\ 4x}$$

$$\lim_{x \to 0} \frac{sen\ 3x}{sen\ 4x} \cdot \frac{1/x}{1/x}$$

$$\lim_{x \to 0} \frac{\frac{sen\ 3x}{x}}{\frac{sen\ 4x}{x}}$$

fazendo 3x=u e 4x=v

$$\lim_{x \to 0} \frac{\frac{sen\ u}{u}}{\frac{sen\ v}{v}}$$

resolvemos .....

$$\lim_{x \to 0} \frac{sen\ 3x}{sen\ 4x} = 3/4$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = \lim_{x\to 0}\frac{\frac{\operatorname{sen} x}{\cos x}}{x}$$

$$= \lim_{x\to 0}\frac{\operatorname{sen} x}{x}\cdot\frac{1}{\cos x}$$

$$= \lim_{x\to 0}\frac{\operatorname{sen} x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}$$

$$= 1\cdot 1$$

$$= 1.$$

$$\lim_{x\to 0}\frac{cos\,x-1}{x}$$

$$\lim_{x\to 0}\frac{cos\,x-1}{x}\cdot\frac{(\cos x+1)}{(\cos x+1)}$$

$$\lim_{x\to 0}\frac{\cos x^2-1}{x\,(\cos x+1)}$$

$$\lim_{x\to 0}\frac{-\operatorname{sen} x^2}{x\,(\cos x+1)}$$

$$-\lim_{x\to 0}\frac{(sen\,x)(sen\,x)}{x\,(\cos x+1)}$$

separando .....

$$-\left(\lim_{x\to 0}\frac{(sen\,x)}{x}\cdot\lim_{x\to 0}\frac{(sen\,x)}{1}\cdot\lim_{x\to 0}\frac{1}{(\cos x+1)}\right)$$

$$-\left(1\,.\,0.\frac{1}{1+1}\right)$$

$$-\left(1\,.\,0.\frac{1}{2}\right)$$

$$\lim_{x\to 0}\frac{cos\,x-1}{x}=0$$