Sistemas Não-Lineares

Método de Newton

Dado um sistema não-linear

$$f_1(x_1, x_2, \ldots, x_n) = 0$$
$$f_2(x_1, x_2, \ldots, x_n) = 0$$
$$\vdots$$
$$f_n(x_1, x_2, \ldots, x_n) = 0$$

O sistema pode ser representado de forma vetorial:

$$F(\underline{x}) = \underline{0}$$

onde: $\underline{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$

Como viu-se no Método de Newton para equações escalares, a cada iteração determina-se a reta tangente ao gráfico da função no ponto inicial. No caso de sistemas de equações, determina-se o hiperplano tangente ao politopo determinado pelo sistemas de equações no ponto inicial. O processo é semelhante ao caso escalar, no qual utiliza-se da expansão em Série de Taylor vetorial no ponto $\underline{x}^{(0)}$.

$$F(\underline{x}) = F(\underline{x}^{(0)}) + J(\underline{x}^{(0)})(\underline{x} - \underline{x}^{(0)})$$

onde:

$$x_{n+1} = x_n - \frac{\begin{vmatrix} F(x,y) & \frac{\partial F}{\partial y} \\ G(x,y) & \frac{\partial G}{\partial y} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix}}$$

$$x_{n+1} = x_n - \frac{F(x,y)\cdot\frac{\partial G}{\partial y} - G(x,y)\cdot\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$y_{n+1} = y_n - \frac{\begin{vmatrix} \frac{\partial F}{\partial x} & F(x,y) \\ \frac{\partial G}{\partial x} & G(x,y) \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix}}$$

$$y_{n+1} = y_n - \frac{G(x,y)\cdot\frac{\partial F}{\partial x} - F(x,y)\cdot\frac{\partial G}{\partial x}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$x_{n+1} = x_n - \frac{F(x,y)\cdot\frac{\partial G}{\partial y} - G(x,y)\cdot\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}} \qquad y_{n+1} = y_n - \frac{G(x,y)\cdot\frac{\partial F}{\partial x} - F(x,y)\cdot\frac{\partial G}{\partial x}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$x^2 + y^2 = 4 \qquad F(x,y) = x^2 + y^2 - 4$$
$$x^2 - y^2 = 1 \qquad G(x,y) = x^2 - y^2 - 1$$

Jacobiano

$$\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ 2x & -2y \end{vmatrix} = -4xy - 4xy = -8xy$$

Este valor precisa ser diferente de zero para o sistema convergir

CHUTE

$$x_0 = 1{,}5$$
$$y_0 = 1{,}5$$

$$-8xy = (-8)(1{,}5)(1{,}5) = -18$$

Converge

$$F(x,y) = x^2 + y^2 - 4$$
$$G(x,y) = x^2 - y^2 - 1$$

$$F(1{,}5;1{,}5) = (1{,}5)^2 + (1{,}5)^2 - 4 = 0{,}5$$
$$G(1{,}5;1{,}5) = (1{,}5)^2 - (1{,}5)^2 - 1 = -1$$

$$\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ 2x & -2y \end{vmatrix} = \begin{vmatrix} 2.(1{,}5) & 2.(1{,}5) \\ 2.(1{,}5) & -2.(1{,}5) \end{vmatrix} = \begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix}$$

$$x_{n+1} = x_n - \frac{F(x,y)\cdot\frac{\partial G}{\partial y} - G(x,y)\cdot\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$x_1 = 1{,}5 - \frac{0{,}5\cdot(-3) - (-1)\cdot 3}{-18} = 1{,}5 - \frac{-1{,}5+3}{-18}$$
$$= 1{,}5 - \frac{+1{,}5}{-18} = 1{,}5 + 0{,}0833 = 1{,}5833$$

$$y_{n+1} = y_n - \frac{G(x,y)\cdot\frac{\partial F}{\partial x} - F(x,y)\cdot\frac{\partial G}{\partial x}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$y_1 = 1{,}5 - \frac{(-1)\cdot(3) - (0{,}5)\cdot(3)}{-18} = 1{,}5 - \frac{-3-1{,}5}{-18}$$
$$= 1{,}5 - \frac{-4{,}5}{-18} = 1{,}5 - 0{,}25 = 1{,}25$$

Cálculo do erro

$$|x_1 - x_0| = |1{,}5833 - 1{,}5| = 0{,}0833$$
$$|y_1 - y_0| = |1{,}25 - 1{,}5| = 0{,}25$$

$$x_1 = 1{,}5833 \qquad -8xy = (-8)(1{,}5833)(1{,}25) = -15{,}833$$
$$y_1 = 1{,}25$$

Converge

$$F(x,y) = x^2 + y^2 - 4$$
$$G(x,y) = x^2 - y^2 - 1$$

$$F(1{,}5833;1{,}25) = (1{,}5833)^2 + (1{,}25)^2 - 4 = 0{,}0693$$
$$G(1{,}5833;1{,}25) = (1{,}5833)^2 - (1{,}25)^2 - 1 = -0{,}0557$$

$$\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ 2x & -2y \end{vmatrix} = \begin{vmatrix} 2.(1{,}5833) & 2.(1{,}25) \\ 2.(1{,}5833) & -2.(1{,}25) \end{vmatrix} = \begin{vmatrix} 3{,}1666 & 2{,}5 \\ 3{,}1666 & -2{,}5 \end{vmatrix}$$

$$x_{n+1} = x_n - \frac{F(x,y)\cdot\frac{\partial G}{\partial y} - G(x,y)\cdot\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$x_2 = 1{,}5833 - \frac{0{,}0693\cdot(-2{,}5) - (-0{,}0557)\cdot(2{,}5)}{-15{,}8}$$
$$= 1{,}5833 - \frac{-0{,}1733 + 0{,}1392}{-15{,}8}$$
$$= 1{,}5833 - \frac{-0{,}0341}{-15{,}8} = 1{,}5833 - 0{,}0022 = 1{,}5811$$

$$y_{n+1} = y_n - \frac{G(x,y)\cdot\frac{\partial F}{\partial x} - F(x,y)\cdot\frac{\partial G}{\partial x}}{\frac{\partial F}{\partial x}\cdot\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\cdot\frac{\partial G}{\partial x}}$$

$$y_2 = 1{,}25 - \frac{(-0{,}0557)\cdot(3{,}1666) - (0{,}0693)\cdot(3{,}1666)}{-15{,}8}$$
$$= 1{,}25 - \frac{-0{,}1763 - 0{,}2196}{-15{,}8}$$
$$= 1{,}25 - \frac{-0{,}3959}{-15{,}8} = 1{,}25 - 0{,}0250 = 1{,}2250$$

Cálculo do erro

$$|x_2 - x_1| = |1{,}5833 - 1{,}5811| = 0{,}0022$$
$$|y_2 - y_1| = |1{,}2500 - 1{,}2250| = 0{,}0250$$